

**MORE master basic syllabus**

**Title:**

*Theoretical and numerical aspects in fluid dynamics and turbulent flows*

**Credit value:**

*3 ECTS*

**Mandatory/Optional:**

*Optional*

**Semester:**

*2*

**Lecturer/s:**

*Luis Vega (UPV/EHU)*

*Francisco De La Hoz (UPV/EHU)*

*Carlos Gorria (UPV/EHU)*

**University:**

*University of the Basque Country*

**Department:**

*Department of Applied Mathematics, Statistics and Operations Research*

*Department of Mathematics*

**Rationale:**

*This course is devoted to the modeling of equations of fluid dynamics in the presence of turbulence, vortices and stochastic flows. The knowledge about these dynamics is essential for the design of durable mechanisms that optimize the capture of energy from either wind or sea forces. Some nonlinear PDE which are susceptible of being studied analytically and numerically are relevant within this context. Moreover, the stochastic Burgers and Navier-Stokes equations are useful in the presence of nondeterministic forces.*

**Objectives:**

*To provide students with a mathematical description of the emergence and propagation of some types of singularities in fluid dynamics. We show the equations modeling these phenomena and we analyze some particular solutions of special interest. Due to the limitations of analytical methods for solving nonlinear partial differential equations and stochastic differential equations that appear in this framework, some sophisticated numerical schemes are proposed. Students will focus on the programming of numerical methods to make an efficient use of them.*

**Skills:** (according to the list of skills provided)

Subject skills	More Master Skills						
	L2.1	L2.2	L2.3	L2.4	L2.5	L2.6	L2.7
L3.1. Learning about the partial differential equations of fluid dynamics, the physical laws that lead into these equations and the assumptions taken in the formulation.			X	X			
L3.2. Knowing the mathematical concepts of vortex filaments, sheets and patches and visualizing the evolution of these types of solutions.	X					X	X
L3.3. Understanding the concept of stochastic forces in fluids and introducing its effect into the equations.		X		X			
L3.4. Being able to program nontrivial numerical schemes to solve partial differential equations with singularities and stochastic differential equations.	X						X

**Teaching and learning methods:**

The subject will be taught principally by master lectures, besides there will be some practical sessions as well as computing practices.

**Allocation of student time:**

	<b>Attendance (classroom, lab,...)</b>	<b>Non attendance (lecture preparation, self study...)</b>
<i>Lectures</i>	<i>22 hours</i>	<i>33 hours</i>
<i>Computer Lab</i>	<i>8 hours</i>	<i>12 hours</i>

**Assessment:**

Home assignments (made individually) 50% and computer programs 50%.

**Assessment Matrix:**

<b>Subject skills</b>	<b>Assessment method</b>		
	<b>Home work</b>	<b>Computer programs</b>	<b>Report</b>
L3.1.	100%		
L3.2.		60%	40%
L3.3.	100%		
L3.4.		60%	40%

**Programme:**

Lesson 1	<p><i>Basic notions of turbulent flows</i></p> <p>From the Euler equations and by taking some assumptions, the formulation can be transformed into some nonlinear PDE suitable for being studied analytically and numerically.</p> <p><i>Distribution (8 h theory + 2 h practical classroom)</i></p>
Lesson 2	<p><i>Vortex filaments, sheets and patches</i></p> <p>The analysis of the self-similar solutions of the PDE involved in fluid dynamics and the numerical simulations help to understand the origin and evolution of singularities.</p> <p><i>Distribution (6 h theory + 4 h computer)</i></p>
Lesson 3	<p><i>Burgers and Stokes equations under stochastic forces</i></p> <p>In the presence of nondeterministic forces the model turns to the stochastic Burgers and Navier-Stokes equations. Design of implicit numerical methods for approximate solutions.</p> <p><i>Distribution (6 h theory + 4 h computer)</i></p>

**Resources:**

*A classroom with a blackboard, laptop and projector for lectures and occasionally a computer room for practical sessions*

**Bibliography:**

Uriel Frisch, *Turbulence, the Legacy of A. N. Kolmogorov*, Cambridge University Press, 1995.

Andrew J. Majda, Andrea L. Bertozzi, *Vorticity and incompressible flow*, Cambridge University Press, 2002.

Philip G. Saffman, *Vortex dynamics*, Cambridge University Press, 1992.

Alexandre J. Chorin, *Vorticity and Turbulence*, Springer, 1994.

Hiroshi Kunita, *Stochastic Flows and Stochastic Differential Equations*, Cambridge University Press, 1990.

Peter E. Kloeden, Eckhard Platen, *Numerical Solution of Stochastic Differential Equations*, Springer, 1999.

Vishik, M.J., Fursikov, A.V., *Mathematical Problems of Statistical Hydromechanics*, Kluwer Academic Publishers, Dordrecht, 1988

**Further comments:**

